WEEKLY TEST OYJ TEST - 21 R & B
SOLUTION Date 08-09-2019

[PHYSICS]

1. Using $d \sin\theta = n\lambda$, for $n = 1$

$$\sin\theta = \frac{\lambda}{d} = \frac{550\times10^{-9}}{0.55\times10^{-3}} = 10^{-3} = 0.001 \text{ rad}$$

2.

By using $\mu = \tan\theta_p \Rightarrow \mu = \tan 60 = \sqrt{3}$,

also $C = \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

3.

$\mu = \tan\theta_P \Rightarrow \theta_P = \tan^{-1} n$

4.

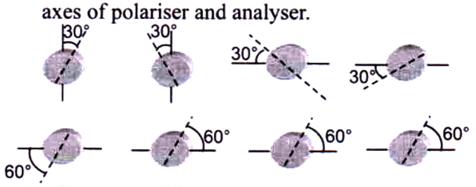
The amplitude will be $A\cos 60° = A/2$

5.

If an unpolarised light is converted into plane polarised light by passing through a polaroid, its intensity becomes half.

6.

Final intensity of light is given by Brewster's law $I = I_0\cos^2\theta$; where θ = Angle between transmission axes of polariser and analyser.



(i) (ii) (iii) (iv)

7.

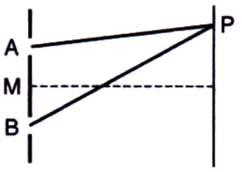
For first minima,

$AP - BP = \lambda$

Hence, $AP - MP = \frac{\lambda}{2}$

$\therefore$ Phase difference

$$= \frac{2\pi}{\lambda}\times\frac{\lambda}{2} = \pi \text{ radian}$$



8.

$$\Delta x = n\lambda$$

or $\quad d\sin\theta = n\lambda \quad$ [For maximum intensity]

For maximum number of possible interference maxima,

$$\sin\theta = 1$$

$\therefore \quad d = n\lambda \quad$ or $\quad 4\lambda = n\lambda \quad$ or $\quad n = 4.$

9.

In Young's double slit experiment intensity at a point is given by:

$$I = I_0\cos^2\left(\frac{\phi}{2}\right)$$

where ϕ = phase difference; I_0 = maximum intensity

or $\quad \frac{I}{I_0} = \cos^2\left(\frac{\phi}{2}\right) \quad$ ...(i)

Phase difference, $\phi = \frac{2\pi}{\lambda}\times$ path difference

$\therefore \quad \phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{6}$

or $\quad \phi = \frac{\pi}{3} \quad$ ...(ii)

Substitute eqn. (ii) in eqn. (i), we get;

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right)$$

or $\quad \frac{I}{I_0} = \frac{3}{4}.$

10.

$$\Delta = x\frac{d}{D}$$

$\therefore$ Phase difference $= \phi = \frac{2\pi}{\lambda}\Delta$

Let a = amplitude at the screen due to each slit

$\therefore \quad I_0 = K(2a)^2 = 4Ka^2,$

where K is a constant.

For phase difference ϕ amplitude $A = 2a\cos(\phi/2)$

Intensity, $\quad I = KA^2 = K(4a)^2\cos^2(\phi/2)$

$$= I_0\cos^2(\pi\Delta/\lambda)$$

$$= I_0\cos^2\left(\frac{\pi}{\lambda}\frac{xd}{D}\right)$$

$$= I_0\cos^2(\pi x/\beta).$$

11.

Path difference $\Delta x = \dfrac{yd}{D}$

Here, $y = \dfrac{d}{2} = \dfrac{5\lambda}{2}$ (as $d = 5\lambda$)

and $D = 10d = 50\lambda$

So, $\Delta x = \dfrac{5\lambda}{2} \times \dfrac{5\lambda}{50\lambda} = \dfrac{\lambda}{4}$

Corresponding phase difference will be,

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$$

or $\dfrac{\Delta\phi}{2} = \dfrac{\pi}{4}$

$\therefore \quad I = I_0 \cos^2\left(\dfrac{\Delta\phi}{2}\right) = I_0 \cos^2\left(\dfrac{\pi}{4}\right) = \dfrac{I_0}{2}.$

12.

$$x_n = 2n\left(\frac{D\lambda}{2d}\right)$$

or $\dfrac{x_n}{D} = \dfrac{n\lambda}{d}$

$\therefore \quad \sin\theta = \dfrac{n\lambda}{d} = \dfrac{3 \times 589 \times 10^{-9}}{0.589}$

or $\theta = \sin^{-1}(3 \times 10^{-6})$.

13.

In first case, $I_{\max.} = (a + a)^2 = 4a^2$

In second case, $I'_{\max.} = a^2 + a^2 = 2a^2$

$\therefore \quad \dfrac{I_{\max.}}{I'_{\max.}} = \dfrac{4a^2}{2a^2} = \dfrac{2}{1}.$

14.

$\phi = 60°, \quad \cos\phi = 1/2, \quad I_1 = I_2 = I_0$

$\therefore \quad I = I_1 + I_2 + 2(\sqrt{I_1 I_2})\cos\phi$

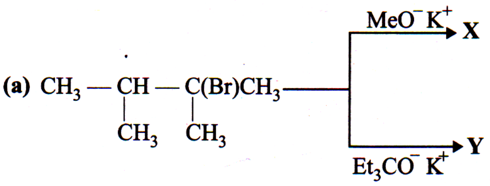
$= I_0 + I_0 + 2(\sqrt{I_0 \times I_0})\cos 60° = 3I_0.$

15.

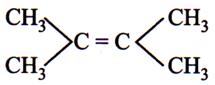
[CHEMISTRY]

16. **(d)**

17. **(a)**

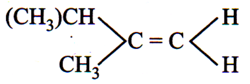


X =



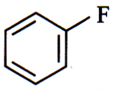
(Saytzeff)

Y =

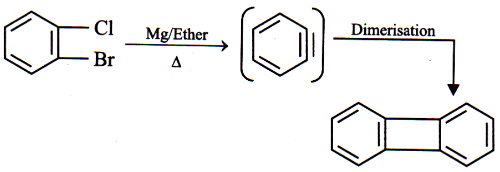


(Hoffmann)

18. **(a)** Fluoro benzene

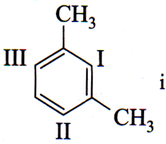


19. **(c)**

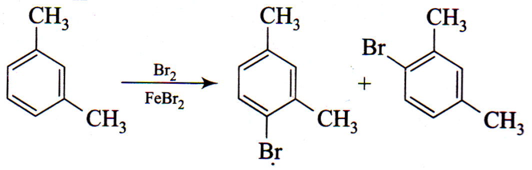


Ortho-dihalobenzene does not form Grignard reagent.

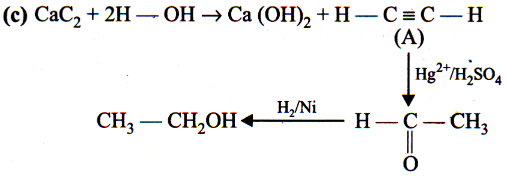
20. **(c)** Methyl group is ortho para directing but due to steric hindrance effect, generated by two CH_3 groups substitution will not take place on position (I). Hence only two products are possible.



i.e.

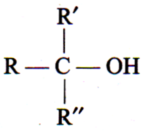


21. **(c)**



$$CaC_2 + 2H-OH \rightarrow Ca(OH)_2 + \underset{(A)}{H-C\equiv C-H} \xrightarrow{Hg^{2+}/H_2SO_4} H-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{H_2/Ni} CH_3-CH_2OH$$

22. **(c)** 3° alcohol is



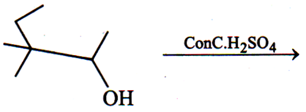
$$R-\overset{\overset{R'}{|}}{\underset{\underset{R''}{|}}{C}}-OH$$

23. **(b)** Tertiary alcohols are formed by treating Grignard reagents either with ketones or excess of an ester other than formate which will give 2° alcohol.

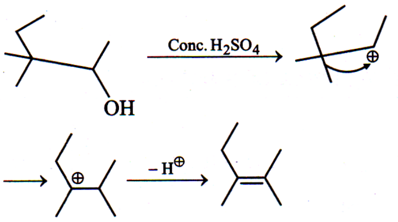
24. **(d)** According to carbocation stability

25. **(d)** Tertiary alcohols react fastest with Lucas reagent followed by 2° and 1° alcohols.

26. **(c)**

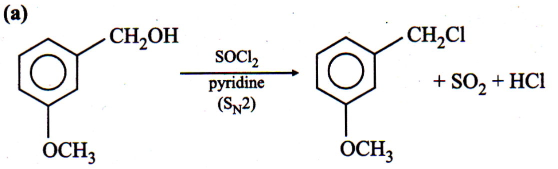


Major product is:



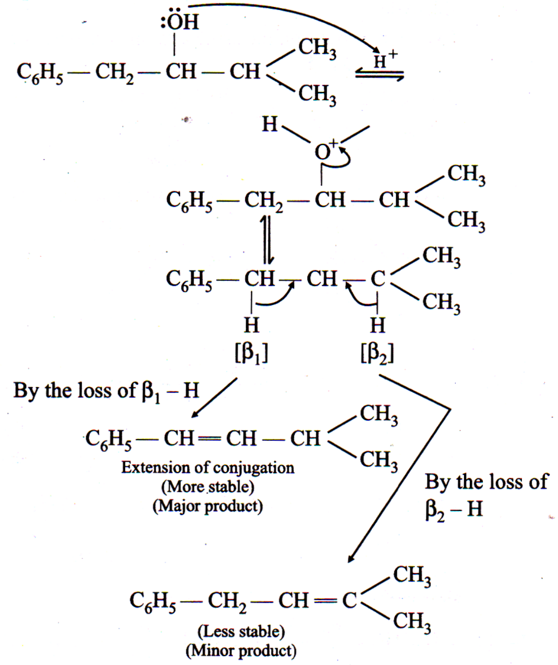
27. **(a)** Nucleophilic substitution of alcohol is acid catalysed reaction.

28. **(a)**

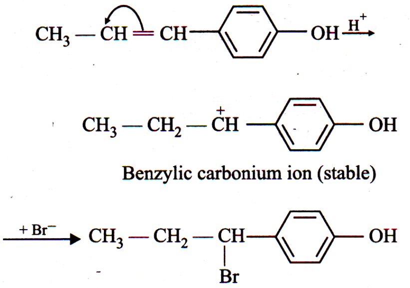


29 **(a)**

$H_2SO_4 \rightleftharpoons H + HSO_4^-$



30. **(b)** The mechanism of this reaction is represented as follows.

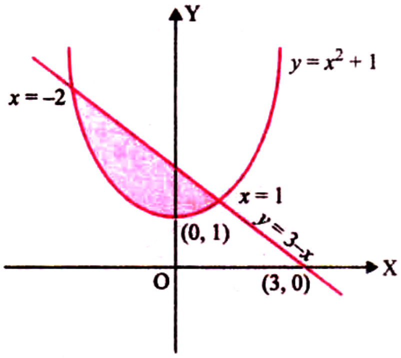


[MATHEMATICS]

31.

The two curves, the parabola and the line meet where $3 - x = x^2 + 1$

$\Leftrightarrow x^2 + x - 2 = 0 \quad \Leftrightarrow x = -2, 1$



Required area is shown shaded in the figure and is equal to

$$\int_{-2}^{1}(y_{\text{line}} - y_{\text{parabola}})\,dx$$

$$=\int_{-2}^{1}\{3-x-(x^2+1)\}\,dx$$

$$=\left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{1}$$

$$=2-\frac{1}{2}-\frac{1}{3}-\left\{-4-2+\frac{8}{3}\right\}=\frac{9}{2}.$$

32.

$$\textbf{Required area} = \int_{1}^{4} 3\sqrt{x}\,dx = 3\left[\frac{x^{3/2}}{3/2}\right]_{1}^{4}$$

$$= 2\,(4^{3/2} - 1) = 2(8 - 1) = 14.$$

33.

Note that in $\left(0, \frac{\pi}{4}\right)$, $\cos x > \sin x$.

$\therefore$ Required area (shown shaded)

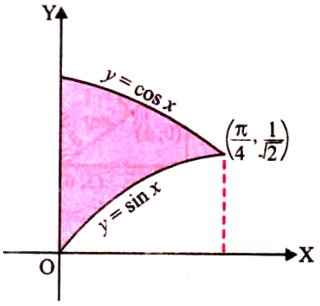


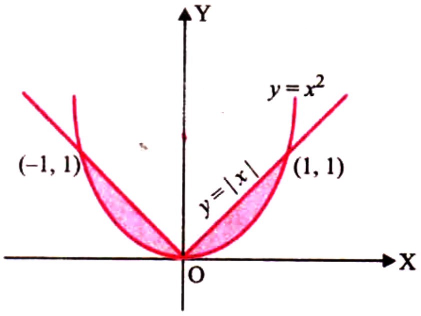
FIGURE 12.20

$$= \int_0^{\pi/4} (\cos x - \sin x)\, dx = [\sin x + \cos x]_0^{\pi/4}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right) = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1.$$

34.

Required area is below the curve $y = | x |$ and above the curve $y = x^2$. It is symmetrical about y-axis. The two curves meet when $x = 0, -1, 1$.



Required area

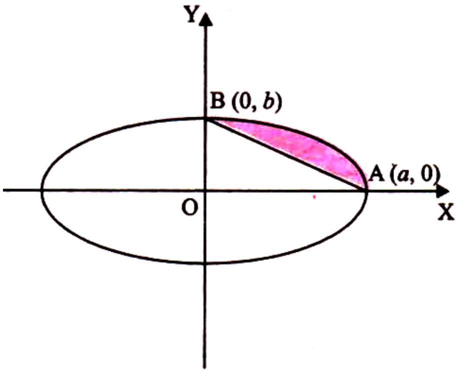
$$= 2\int_0^1 (| x | - x^2)\, dx = 2\int_0^1 (x - x^2)\, dx.$$

35.

Ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \quad \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$\Rightarrow \quad y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$$



Now area of $\Delta AOB = \frac{1}{2} | OA | \times | OB |$

$$= \frac{1}{2} ab \text{ sq. units}$$

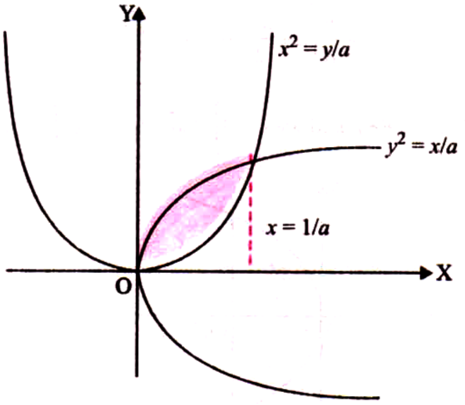
and area of the ellipse in the first quadrant

$$= \int_0^a y\, dx = \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx$$

36.

The two curves met where

$x = a\,(ax^2)^2 \Rightarrow x = a^3x^4 \Rightarrow x = 0,\ \frac{1}{a}$



$$\therefore \text{ Area in reference } = \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx$$

$$\Rightarrow 1 = \frac{1}{\sqrt{a}}\frac{2}{3}\left[x^{3/2}\right]_0^{1/a} - \left[\frac{ax^3}{3}\right]_0^{1/a}$$

$$\Rightarrow 1 = \frac{2}{3}\frac{1}{\sqrt{a}\,a^{3/2}} - \frac{a}{3}\left(\frac{1}{a^3}\right) \Rightarrow 1 = \frac{1}{3a^2}$$

$$\Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \frac{1}{\sqrt{3}}.$$

37.

$$\text{Required area} = \int_{1-e}^{0}\log(x+e)\,dx$$

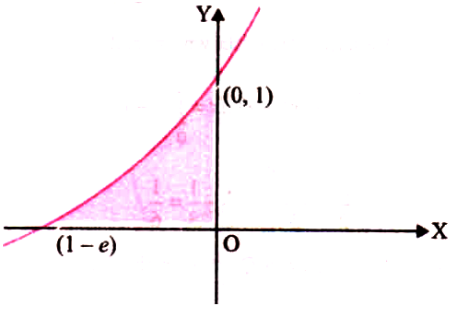
($y = \log_e(x+e)$) meets x-axis where $y = 0$

$\Rightarrow \log(x+e) = 0 \Rightarrow x + e = 1$

$\Rightarrow x = 1 - e$

Also it meets y-axis where $x = 0$

i.e., where $y = \log_e e = 1$)



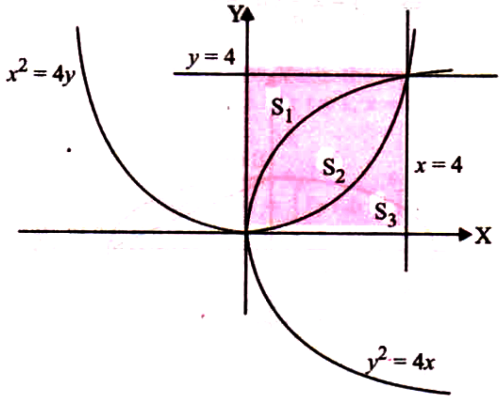
$$= \left[x\log(x+e)\right]_{1-e}^{0} - \int_{1-e}^{0}\frac{1}{x+e}\,x\,dx$$

$$= 0 - \int_{1-e}^{0}\frac{x+e-e}{x+e}\,dx = -\left[x - e\log(x+e)\right]_{1-e}^{0}$$

$$= -\{0 - e - (1-e)\} = 1.$$

38.

$$= 16 - \frac{4\times4^{3/2}}{3} = 16 - \frac{32}{3} = \frac{16}{3}.$$



$$S_2 = \int_0^4\left[2\sqrt{x} - \frac{x^2}{4}\right]_0^4 = \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \frac{4}{3}\times4^{3/2} - \frac{16}{3} = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

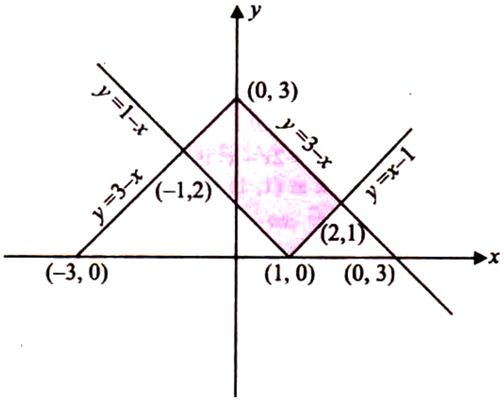
$$\text{and } S_3 = \int_0^4\left(\frac{x^2}{4}\right)dx = \left[\frac{x^3}{12}\right]_0^4 = \frac{16}{3}.$$

$\therefore\ S_1 : S_2 : S_3 : 1 : 1 : 1.$

39.

40.

Required area is shown shaded in the figure.



The two curves meet where

$3 - |x| = |x - 1| \Rightarrow |x| + |x - 1| = 3$

$\Rightarrow x = -1, 2$ (Solve yourself)

Required area $= \int_{-1}^{2} (3-|x|-|x-1|)\,dx$

$$= \left[3x - \frac{x|x|}{2} - \frac{(x-1)|x-1|}{2}\right]_{-1}^{2}$$

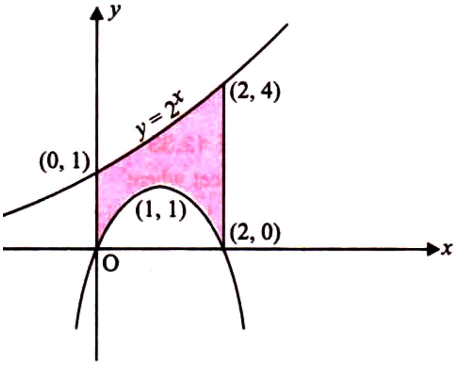
$$= \left(6-2-\frac{1}{2}\right)-\left(-3+\frac{1}{2}+2\right)=4.$$

41.

The curve $y = 2x - x^2$ is a downward parabola with vertex at (1, 1)

$\{y = 2x - x^2 \Leftrightarrow y - 1 = -(x-1)^2$

$\Leftrightarrow (x-1)^2 = -(y-1)\}$



The line $x = 2$ meets this parabola where $y = 4 - 4 = 0$, *i.e.*, in the point (2, 0) and the curve $y = 2^x$ in the point (2, 4).

Required area in shown shaded in the figure and is equal to

$$\int_0^2 [2^x - (2x - x^2)]\,dx = \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$$

$$= \frac{3}{\log 2} - 4 + \frac{8}{3} = \frac{3}{\log 2} - \frac{4}{3}.$$

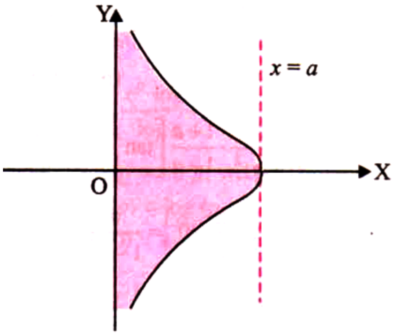
42.

Given curve is $xy^2 = a^2(a - x)$

which meets x-axis where $y = 0$, *i.e.*,

where $x = a$. Also, $y^2 = a^2\left(\frac{a-x}{x}\right) \geq 0$

$$\Rightarrow \frac{a-x}{x} \geq 0 \Rightarrow (a-x)\,x \geq 0,\ x \neq 0$$



$$\Rightarrow x\,(x-a) \leq 0,\ x \neq 0 \Rightarrow 0 < x \leq a.$$

The curve does not meet y-axis and lies between the lines $x = a$ and $x = 0$.

Note that the curve is symmetrical about x-axis. ($\because$ if y is changed to $-y$, the equation remains unchanged)

Also, we find that (on differentiation)

$$2y\frac{dy}{dx} = a^2\left(-\frac{a}{x^2}\right) \Rightarrow \frac{dy}{dx} = -\frac{a^3}{2x^2y}.$$

$\therefore$ At $(a, 0)$, there is a vertical tangent.

Required area = 2 × area in the first quadrant.

$$= 2\int_0^a |\,y\,|\,dx = 2\int_0^a a\sqrt{\frac{a-x}{x}}\,dx$$

Substitute $\sin^{-1}\sqrt{\frac{x}{a}} = \theta$, *i.e.*, $x = a\sin^2\theta$

$$= 2\int_0^{\pi/2} a\sqrt{\frac{a - a\sin^2\theta}{a\sin^2\theta}}\,(2a\sin\theta\cos\theta)\,d\theta$$

$$= 4a^2\int_0^{\pi/2}\cos^2\theta\,d\theta = 4a^2\left(\frac{\pi}{4}\right) = \pi a^2.$$

43.

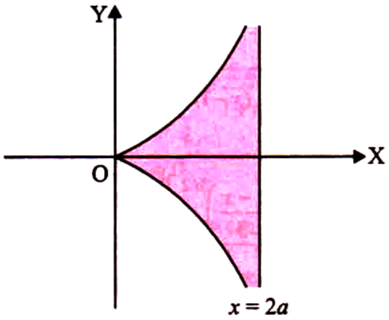
Given curve is $y^2(2a-x) = x^3$

which meets the axes only at the origin. Also,

$$y^2 = \frac{x^3}{2a-x} = x^2\left(\frac{x}{2a-x}\right) \geq 0$$

$$\Rightarrow \frac{x}{2a-x} \geq 0,\ x \neq 2a$$

$$\Rightarrow x\,(2a-x) \geq 0,\ x \neq 2a \Rightarrow 0 \leq x < 2a$$



So, the curve lies between $x = 0$ and $x = 2a$.

Note that the curve is symmetrical about x-axis. ($\because$ if y is changed to $-y$, the equation remains unchanged)

Also, we find that (on differentiation)

$$2y\frac{dy}{dx} = \frac{(2a-x)\,3x^2 - x^3\,(-1)}{(2a-x)^2} = \frac{x^2\,\{6a-2x\}}{(2a-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2\,(a-3x)}{(2a-x)^2\,y}$$

$\therefore$ At $(0, 0)$, slope of tangent = 0

Required area $= 2\int_0^{2a} |\,y\,|\,dx = 2\int_0^{2a}\sqrt{\frac{x^3}{2a-x}}\,dx$

substitute $\sin^{-1}\sqrt{\frac{x}{2a}} = \theta$,

i.e., $x = 2a\sin^2\theta \Rightarrow dx = 4a\sin\theta\cos\theta\,d\theta$

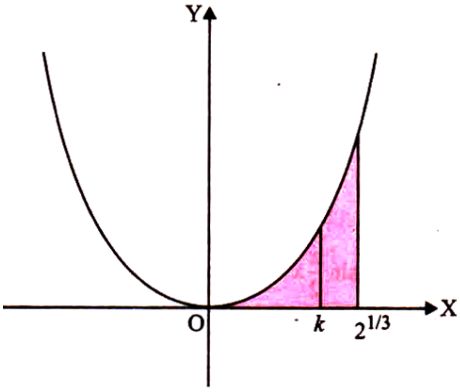
$$= 2\int_0^{\pi/2}\sqrt{\frac{8a^3\sin^6\theta}{2a\cos^2\theta}}\,4a\sin\theta\cos\theta\,d\theta$$

$$= 8a\int_0^{\pi/2} 2a\sin^4\theta\,d\theta = 16a^2\int_0^{\pi/2}\sin^4\,d\theta$$

$$= 16a^2\left(\frac{3\times 1}{4\times 2}\times\frac{\pi}{2}\right) = 3\pi a^2.$$

44.

$$\int_0^k x^2\,dx = \int_k^{2^{1/3}} x^2\,dx \Rightarrow \frac{k^3}{3} = \frac{2}{3} - \frac{k^3}{3}$$



$$\Rightarrow 2k^3 = 2 \Rightarrow k^3 = 1 \Rightarrow k = 1.$$

45.

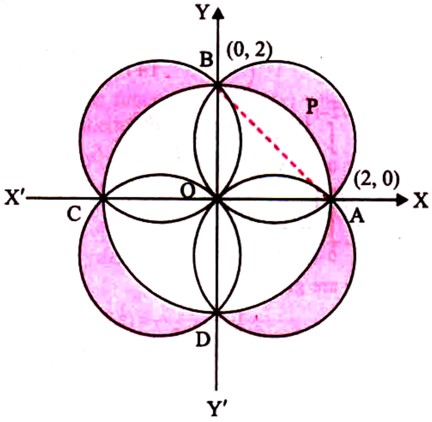
The equation $x^2 + y^2 = 4$ represents a circle with centre at (0, 0) and radius 2. The equation

$$x^2 + y^2 - 2|x| - 2|y| = 0$$

represents four circles

$$x^2 + y^2 - 2x - 2y = 0,$$
$$x^2 + y^2 + 2x - 2y = 0,$$
$$x^2 + y^2 + 2x + 2y = 0 \text{ and}$$
$$x^2 + y^2 - 2x + 2y = 0,$$

each of radius $\sqrt{2}$ and having centres at (1, 1), (–1, 1), (–1, –1) and (1, –1) respectively. The area in reference is outside the circle $x^2 + y^2 = 4$ and inside the four circles. This is shown shaded in the figure.



It is symmetrical in all the four quadrants. In the first quadrant it is bounded by the circles

$$x^2 + y^2 = 4 \text{ and } x^2 + y^2 - 2x - 2y = 0$$

Required area

= 4 × (Area shown shaded in the first quadrant)

$$= 4\left\{\left(\frac{1}{2}\text{ area of small circle}\right) - (\text{area of segment APB})\right\}$$

$$= 4\left\{\frac{1}{2}\pi(\sqrt{2}\text{ unit})^2 - \frac{1}{4}\text{ area of big circle} - \text{area }\Delta AOB\right\}$$

$$= 4\left\{\pi - \left(\frac{1}{4}\pi\times 2^2 - \frac{2\times 2}{2}\right)\right\} \text{ square units.}$$